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Reliability Studies for the Nuclear-Powered Artificial Heart Program

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RELIABILITY STUDIES

FOR

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ABSTRACT

By assuming that the failures of an artificial heart system with a mean life of 10 years can be modeled by a particular probability distribution, both the probability of a failure in the system within t years and the reliability required of each subsystem and component are investigated.

1. INTRODUCTION

The objective of the Nuclear-Powered Artificial Heart Prototype System Development Program being conducted by the Division of Biomedical Environmental Research (DBER) of the Energy Research and Development Administration (ERDA) is to develop a fully implantable nuclear-powered artificial heart with a mean life (life expectancy) of 10 yr. This report investigates the reliability of such an artificial heart system. Here, reliability is defined as the probability that a device will perform adequately for a specified period after implantation. It also indicates the implications of various assumptions about subsystem or component reliability and the requirements that may be imposed on component structure in light of DBER's objectives.

For simplicity, the heart is treated first as a single basic unit. By assuming that the artificial heart system failures can be modeled by a particular probability distribution, the probability of a failure in the system within 1, 2, ..., 15 yr is calculated. In particular, the exponential, normal, lognormal, gamma, and Weibull distributions are considered.

In reality, however, the artificial heart is composed of 4 subsystems with 53, 37, 5, and 4 components, respectively.[1] Using the probability distributions mentioned previously and further assuming that the heart is a simple series system each of whose subsystems and components has the same failure distribution, the mean life required of each subsystem and, subsequently, of each component is calculated. In this calculation, however, it is necessary to alter DBER's objective of developing a heart with a mean life of 10 yr by considering a heart with a median life of 10 yr instead. (Discussion of this topic begins on page 10.) Given results whose distribution is unknown, the additional constraint of setting equal to 0.5 the probability that the system's time to failure will be less than or equal to 10 yr, i.e., $P(T \leq 10) = 0.5$, makes 10 yr the median of the distribution by definition.

Finally, two other approaches to this problem are considered. If each heart system component is assumed to have the same failure distribution, the median life required of each subsystem with k_i components can be calculated. Further, if each subsystem is assumed to have the same failure distribution, in particular, the same median life, the mean life required of each component of a particular subsystem also can be calculated. These results are tabulated for closer inspection.

This report does not give any final answers about the reliability and mean or median life requirements of the components of a complex artificial heart system like the prototype that DBER is developing. It does indicate some possible approaches and their consequences by simplifying the problem through easing considerations and calculations.

2. THE HEART AS A SINGLE BASIC SYSTEM

2.1. The Exponential Failure Distribution

The exponential distribution is chosen as the failure distribution if it can be assumed that the failure rate function is constant, say λ . This assumption implies a lack-of-memory property or no "aging" effect.

Let T be a random variable that represents system time to failure. The exponential probability density function, pdf, is

$$f(t) = \begin{cases} \frac{1}{\lambda} e^{-t/\lambda}, & t > 0 \\ 0 & ,\text{elsewhere}, \end{cases}$$

where t denotes time. The mean, E(T), is λ which when set equal to 10 yr, the mean life of the heart system, implies that the variance is $V(T) = \lambda^2 = 100$. The standard deviation or positive square root of the variance is SD(T) = 10. To calculate the probability of a failure in the system within t years, use

$$P(T \leq t) = \int_0^t \frac{1}{\lambda} e^{-x/\lambda}\, dx = 1 - e^{-t/\lambda},$$

where $P(T \leq t)$ often is denoted by F(t), the (cumulative) distribution function or cdf. Figures 1-3 are graphs of the exponential pdf, cdf, and failure rate function, respectively.

The probabilities of $\lambda = 10$ and t = 1, 2, ..., 15 are summarized to four decimal places in Table I (at the end of this report). Note that when t = 7 yr, the probability of a failure in the system is 0.5034 and when t = 10 yr, the probability is 0.6321. For the probability of a failure within 10 yr to be about 0.5, a mean life of 14.4 yr is required.

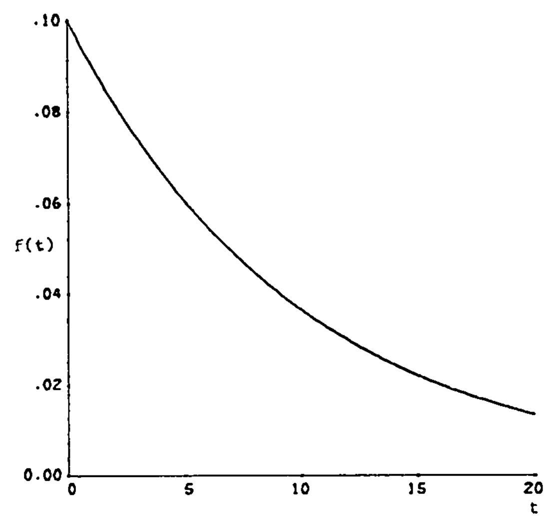

Fig. 1. The exponential probability density function with $\lambda = 10$.

2.2. The Normal Failure Distribution

If the system is subject to aging or gradual failure of its electrical or mechanical components, a normal distribution may be useful in characterizing the failure distribution.

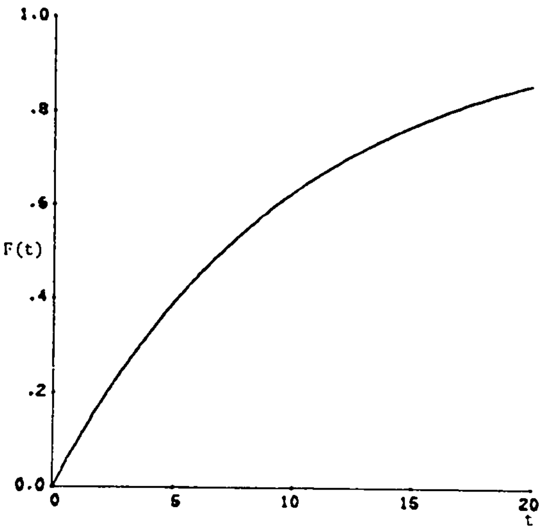

Fig. 2. The exponential cumulative distribution function with $\lambda = 10$.

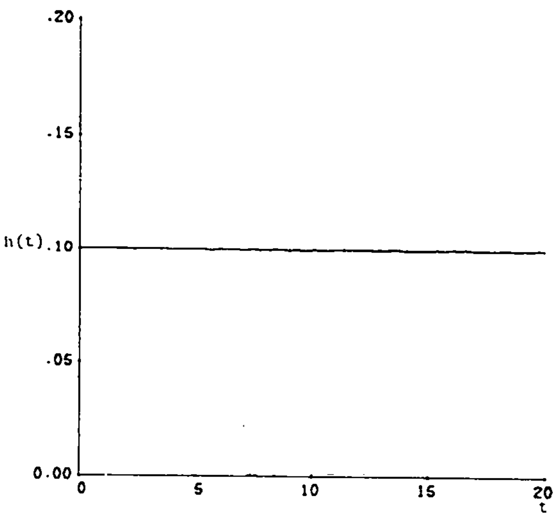

Fig. 3. The exponential failure rate function with $\lambda = 10$.

The normal pdf is

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right], \quad -\infty < t < \infty,$$

where μ, σ^2, and σ are the mean, variance, and standard deviation, respectively. If a change of variable, $z = (t-\mu)/\sigma$, is used, the probability of a failure in the system within t years is given by

$$P(T \leq t) = P\left(Z \leq \frac{t-\mu}{\sigma}\right)$$

$$= \int_{\infty}^{\frac{t-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz = \Phi\left(\frac{t-\mu}{\sigma}\right) .$$

The probabilities for this standardized normal distribution with mean zero and variance one (standard deviation one) are conveniently tabulated and available in most statistics texts.[2] As two parameters, μ and σ, are involved, set $\mu = 10$ and select various and arbitrary values of σ to investigate the quantity $P(T \leq t)$. Tables II-VI exhibit the calculations for $\mu = 10$ and $\sigma = 1, 3, 5, 7$, and 10, respectively. The normal pdf, cdf, and failure rate function for these five sets of parameter values are shown in Figs. 4-6.

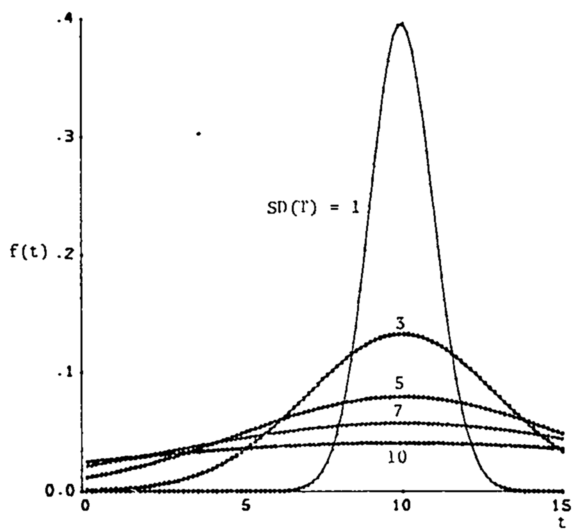

Fig. 4. The normal probability density function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

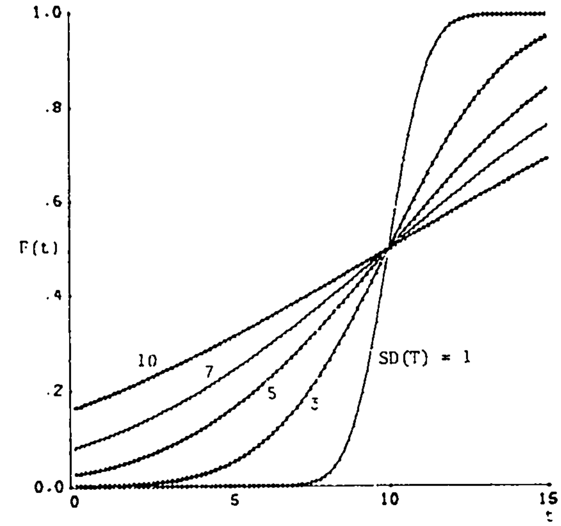

Fig. 5. The normal cumulative distribution function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

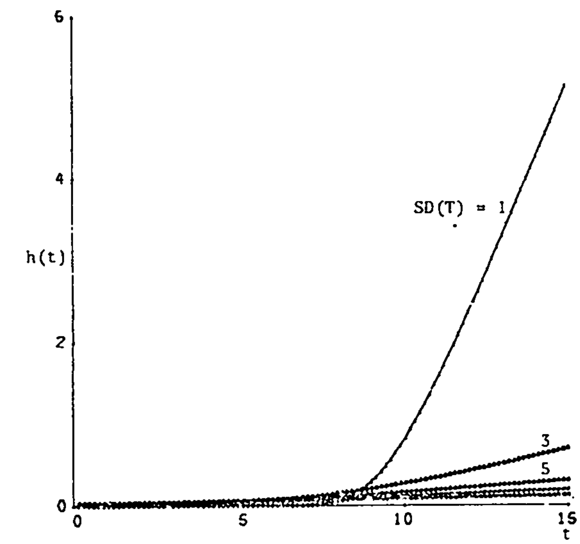

Fig. 6. The normal failure rate function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

2.3. The Lognormal Failure Distribution

If $X = \log_e T$ is normally distributed, T is said to have a lognormal distribution. This distribution has been used to describe the distribution of nuclear reactor failure rates. The pdf, mean, and variance are

$$f(t) = \begin{cases} \dfrac{1}{t\sqrt{2\pi}\,\sigma} \exp\left[-\,(\log_e(t)-\mu)^2/(2\sigma^2)\right], & t > 0 \\ 0 & , \text{ elsewhere} \end{cases},$$

$$E(T) = e^{\mu+\sigma^2/2} ,$$

and

$$V(T) = e^{2\mu}\, e^{\sigma^2}\, (e^{\sigma^2}-1) .$$

For E(T) = 10 and V(T) = 1, say,

$$\mu = E(X) = \log_e 10 - \sigma^2/2$$

and

$$\sigma^2 = V(X) = \log_e(1.01) \sim 0.01 .$$

The standard deviation is $SD(X) \sim 0.10$. By substitution, $\mu = \log_e 10 - 0.01/2 \sim 2.30$. These μ and σ^2 values can then be used in calculating the probability of a failure in the system within t years, where

$$P(T \le t) = P(X \le \log_e t) = P\left(Z \le \frac{\log_e t-\mu}{\sigma}\right)$$

$$= \int_{-\infty}^{\frac{\log_e t-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\, dz$$

$$= \Phi\left(\frac{\log_e t-\mu}{\sigma}\right),$$

which is tabulated as previously mentioned.

When other values for the standard deviation of T are assigned and calculations similar to those shown above are performed, E(T) = 10 and SD(T) = 3 imply that $\mu \sim 2.26$ and $\sigma = 0.29$; E(T) = 10 and SD(T) = 5 imply that $\mu \sim 2.19$ and $\sigma = 0.47$; E(T) = 10 and SD(T) = 7 imply that $\mu \sim 2.10$ and $\sigma = 0.63$; and E(T) = 10 and SD(T) = 10 imply that $\mu \sim 1.96$ and $\sigma = 0.83$. Results are shown in Tables II-VI.

The lognormal pdf, cdf, and failure rate function for the five cases considered above are shown in Figs. 7-9.

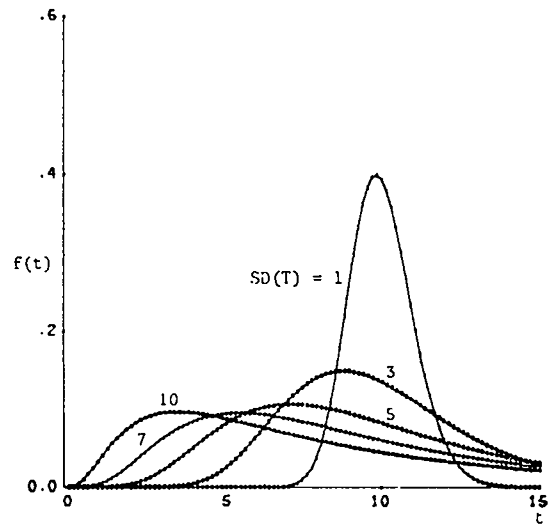

Fig. 7. The lognormal probability density function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

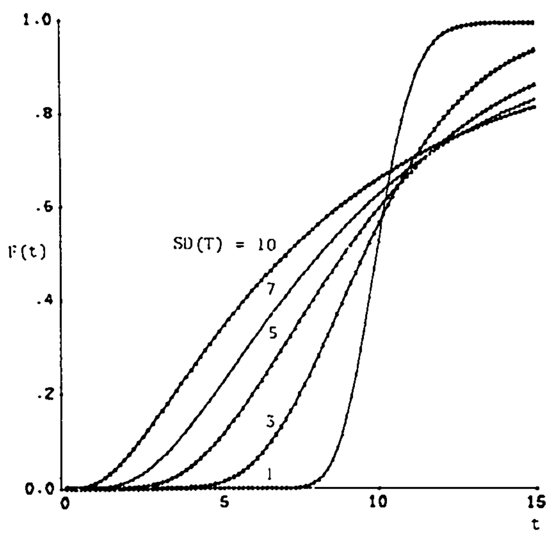

Fig. 8. The lognormal cumulative distribution function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

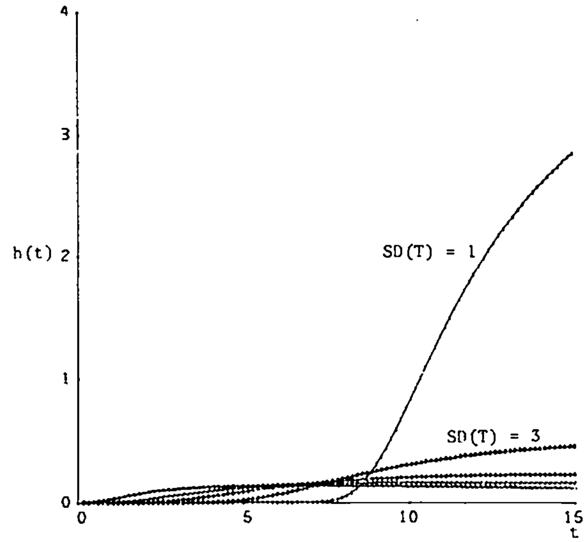

Fig. 9. The lognormal failure rate function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

2.4. The Gamma Failure Distribution

The gamma distribution may be useful in characterizing failures if the components in a complex electromechanical system fail instantaneously during the initial (burn-in) stage or the wear-out period of operation.

The gamma pdf, mean, and variance are

$$f(t)=\begin{cases} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} t^{\alpha-1} e^{-t/\beta}, & 0<t<\infty \\ 0 & ,\text{elsewhere}, \end{cases}$$

$$E(T) = \alpha\beta,$$

and

$$V(T) = \alpha\beta^2,$$

where α is the shape parameter and β is the scale parameter. Note that when $\alpha = 1$, the gamma pdf reduces to the exponential pdf. By graphical depiction, the failure intensity rate increases for $\alpha > 1$ and decreases for $\alpha < 1$.

The chi-square distribution is a special case of the gamma distribution with $\alpha = \nu/2$ and $\beta = 2$ where ν is a positive integer representing the degrees of freedom.

If T has a gamma distribution with parameters α and β, then there exists an X such that $T = \frac{\beta X}{2}$ where the random variable X has a chi-square distribution with $\nu = 2\alpha$ degrees of freedom. Thus, the probability of a failure within t years can be expressed as

$$P(T \leq t) = P(X \leq 2t/\beta).$$

To calculate that probability, three approximations may be used.[3] The first approximation standardizes the random variable X, that is, subtracts its mean, ν, and divides by its standard deviation, $\sqrt{2\nu}$. As $\nu \to \infty$, the standardized chi-square distribution approaches the standardized normal distribution. Thus,

$$P(X \leq 2t/\beta) \sim \Phi\left[\left(\frac{2t}{\beta} - \nu\right)(2\nu)^{-1/2}\right]$$

$$= \Phi\left[\left(\frac{2t}{\beta} - 2\alpha\right)(2(2\alpha))^{-1/2}\right]$$

$$= \Phi\left(\frac{\frac{t}{\beta} - \alpha}{\sqrt{\alpha}}\right). \quad (1)$$

The second approximation is known as Fisher's approximation, and the third is the Wilson-Hilferty approximation. Both use approximate standardization and are given as

$$P(X \leq 2t/\beta) \sim \Phi\left[\sqrt{2(2t/\beta)} - \sqrt{2(2\alpha) - 1}\right]$$

$$= \Phi\left(2\sqrt{t/\beta} - \sqrt{4\alpha - 1}\right) \quad (2)$$

$$P(X \leq 2t/\beta) \sim \Phi\left\{\left[\left(\frac{2t}{\beta(2\alpha)}\right)^{1/3} - 1 + \frac{2}{9}(2\alpha)^{-1}\right]\sqrt{9\left(\frac{2\alpha}{2}\right)}\right\}$$

$$= \Phi\left\{\left[\left(\frac{t}{\alpha\beta}\right)^{1/3} - 1 + \frac{1}{9\alpha}\right] 3\sqrt{\alpha}\right\}. \quad (3)$$

Of these three approximations, the first is the least accurate and the third is the most accurate, unless 2α is large, in which case, the difference in accuracy among them is small.

Again, through the arbitrary and consistent assignment of SD(T) = 1, 3, 5, 7, and 10, calculations for these five cases using the Wilson-Hilferty approximation are shown in Tables II-VI, respectively. Setting E(T) = 10 and SD(T) = 1 gives $\alpha = 100$ and $\beta = 0.10$. Similarly, for E(T) = 10 and SD(T) = 3, $\alpha \sim 11.11$ and $\beta = 0.90$; for E(T) = 10 and SD(T) = 5, $\alpha = 4$ and $\beta = 2.50$; for E(T) = 10 and SD(T) = 7, $\alpha \sim 2.04$ and $\beta = 4.90$; and for E(T) = 10 and SD(T) = 10, $\alpha = 1$ and $\beta = 10$.

Figures 10-12 show the gamma pdf, cdf, and failure rate function.

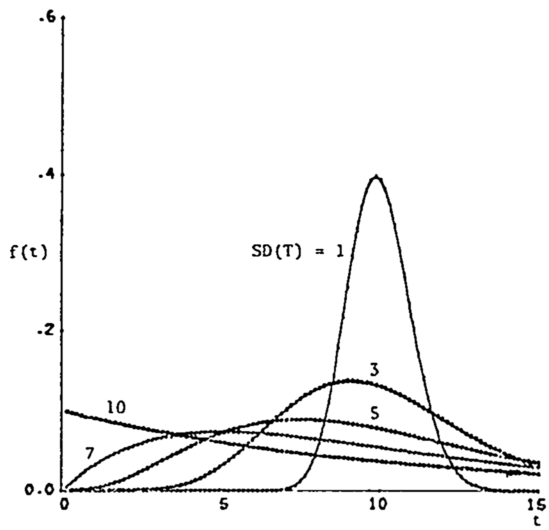

Fig. 10. The gamma probability density function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

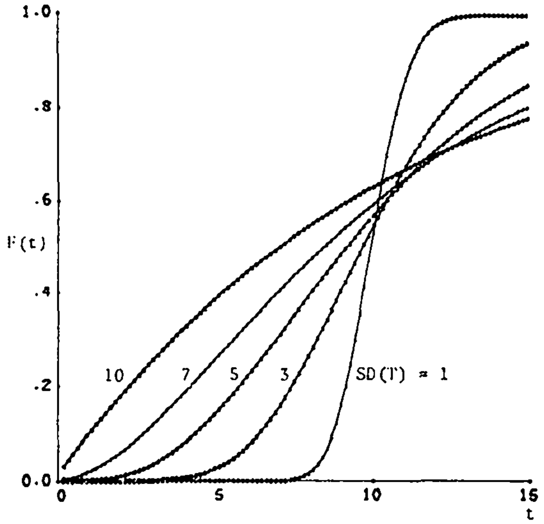

Fig. 11. The gamma cumulative distribution function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

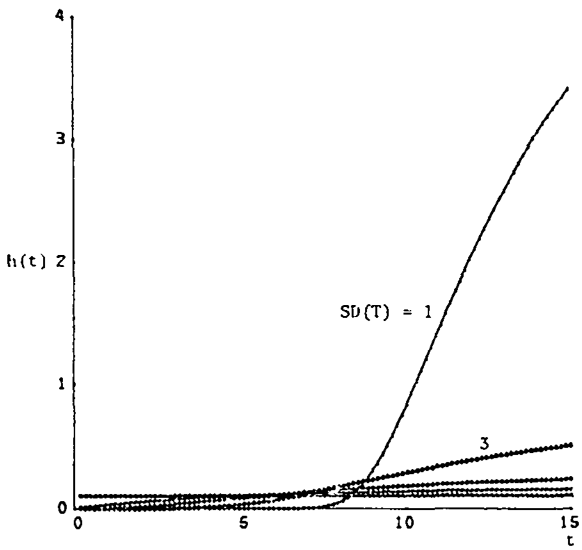

Fig. 12. The gamma failure rate function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

2.5. The Weibull Failure Distribution

The Weibull distribution has been useful in accelerated testing of components operating under forced conditions, testing of equipment such as ball bearings and electron tubes during the initial-failure phase, and in a system involving several components whose failure is attributed to the severest flaw.

The Weibull pdf is

$$f(t)=\begin{cases}\frac{\gamma}{\beta}\, t^{\gamma-1} \exp\left(-\frac{t^{\gamma}}{\beta}\right), & t \geq 0 \\ 0 & \text{, elsewhere,}\end{cases}$$

where γ is the shape parameter and β is the scale parameter. When $\gamma = 1$, the Weibull pdf becomes the exponential pdf. An exponential-type graph which can be used to characterize a decreasing failure intensity rate results when $\gamma < 1$. Also, Weibull pdf's with $\gamma < 1$ are useful in describing catastrophic failures. When $\gamma > 1$, the graph of the Weibull pdf is unimodal. In a more general case, wear-out failures and increasing failure intensity rates are best characterized using $\gamma > 1$ and $t > \alpha > 0$. The mean and variance of the Weibull distribution are given by

$$E(T) = \beta^{1/\gamma}\Gamma(1 + 1/\gamma),$$

$$V(T) = \beta^{2/\gamma}[\Gamma(1 + 2/\gamma) - \Gamma^2(1 + 1/\gamma)].$$

To calculate the probability of a failure within t years, the values of γ and β must be determined. For E(T) = 10 and V(T) = 1, say,

$$\beta^{2/\gamma} = \frac{100}{\Gamma^2(1 + 1/\gamma)},$$

and

$$\frac{\Gamma(1 + 2/\gamma)}{\Gamma^2(1 + 1/\gamma)} = 1.01 \quad .$$

In a trial and error process, $\gamma \sim 12.15$ and $\beta \sim 2.38 \times 10^{12}$ are a solution to the above equations. In similar calculations, E(T) = 10 and SD(T) = 3 give $\gamma \sim 3.71$ and $\beta \sim 7.57 \times 10^3$; E(T) = 10 and SD(T) = 5 give $\gamma \sim 2.10$ and $\beta \sim 1.63 \times 10^2$; E(T) = 10 and SD(T) = 7 give $\gamma \sim 1.45$ and $\beta \sim 32.59$; and E(T) = 10 and SD(T) = 10 give $\gamma \sim 1.00$ and $\beta \sim 10$. Note the large β values that result. Such parameter values seem very unrealistic.

Figures 13-15 are graphs of the Weibull pdf, cdf, and failure rate function for each of the five cases.

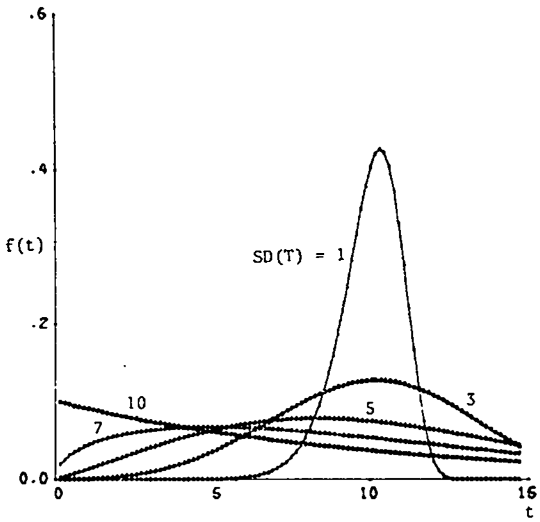

Fig. 13. The Weibull probability density function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

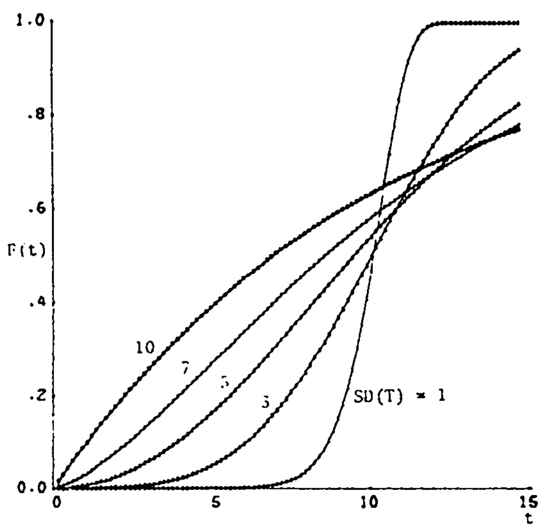

Fig. 14. The Weibull cumulative distribution function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

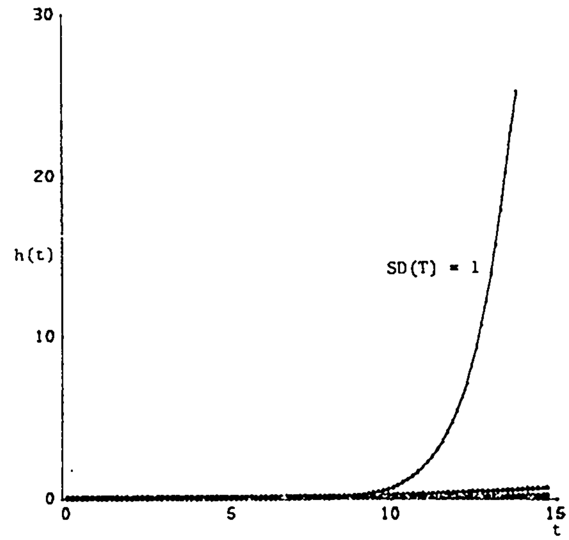

Fig. 15. The Weibull failure rate function with E(T) = 10 and SD(T) = 1, 3, 5, 7, and 10.

The probability of a failure within t years is given by

$$P(T \leq t) = \int_0^t \gamma/\beta \; x^{\gamma-1} \exp(- x^{\gamma}/\beta) \; dx$$

$$= 1 - \exp(-t^{\gamma}/\beta) \quad .$$

The results are shown in Tables II-VI.

2.6. Summary: The Probability of a Failure in the Heart System

For the exponential distribution, the probability of a failure in the heart system is high during the initial years, reaches 0.5 before the seventh year instead of the tenth year, and does not exceed 0.8 even during the fifteenth year.

The results for the other distributions considered (see Tables II-VI) show similar probabilities within tables. In a comparison among tables, the probabilities of a failure during the first 10 years generally increase as the standard deviations increase, then after the tenth year the probabilities generally decrease as the standard deviations increase.

3. A HEART COMPOSED OF 4 SUBSYSTEMS AND 99 COMPONENTS

The four subsystems of the artificial heart are a thermal converter, a blood pump, a flexible shaft assembly, and a cooling system.[1] It will be assumed that they are connected in series; that is, failure of any one subsystem implies failure of the entire heart system, where each subsystem operates independently of the others (Appendix A).

3.1. The Exponential Failure Distribution

First, consider the case in which each subsystem has an exponential failure distribution with parameter λ. The probability of a failure in the system within t years is

$$P(T_S \leq t)$$
$$= P\text{ (at least one subsystem fails before time t)}$$
$$= 1 - P(SS_1 > t,\ SS_2 > t,\ SS_3 > t,\ SS_4 > t)$$
$$= 1 - P(SS_1 > t)\ P(SS_2 > t)\ P(SS_3 > t)\ P(SS_4 > t)$$
$$= 1 - (e^{-t/\lambda})^4 = 1 - e^{-4t/\lambda} \quad ,$$

where S refers to system and SS_n refers to subsystem n. The result is an exponential distribution with parameter $\lambda/4$. The mean and variance of the random variable T for the system are

$$E(T_S) = \lambda/4 = 10 \Longrightarrow \lambda = 40,$$

and

$$V(T_S) = 1600 \quad .$$

Thus, if each subsystem has a mean life of 40 yr and a variance of 1600 yr (or a standard deviation of 40 yr), the heart system will have a mean life of 10 yr and a variance of 100 yr (or a standard deviation of 10 yr). From a practical standpoint, such large variances are undesirable. Moreover, a mean life of 40 yr is not realistic.

To go one step further, assume that the four subsystems have k_1, k_2, k_3, and k_4 components, respectively. Each component has an exponential failure distribution with parameter λ and each operates independently. In reality, several different failure distributions may be represented within a subsystem and failure of a single component may not imply failure of the subsystem and, subsequently, of the entire heart system as the series connection suggests. Nevertheless, for simplicity of calculation and illustration, these assumptions are made.

The probability of a failure in the heart is now given as

$$P(T_S \leq t)$$
$$= P\text{ (at least one subsystem fails before time t)}$$
$$= 1 - P(SS_1 > t,\ SS_2 > t,\ SS_3 > t,\ SS_4 > t)$$
$$= 1 - P(SS_1 > t)\ P(SS_2 > t)\ P(SS_3 > t)\ P(SS_4 > t)$$
$$= 1 - P(C_{11} > t, \ldots, C_{1k_1} > t)\ P(C_{21} > t, \ldots, C_{2k_2} > t)\ P(C_{31} > t, \ldots, C_{3k_3} > t)\ P(C_{41} > t, \ldots, C_{4k_4} > t)$$
$$= 1 - P(C_{11} > t) \ldots P(C_{1k_1} > t)\ P(C_{21} > t) \ldots P(C_{2k_2} > t)\ P(C_{31} > t) \ldots P(C_{3k_3} > t)\ P(C_{41} > t) \ldots P(C_{4k_4} > t)$$
$$= 1 - (e^{-t/\lambda})^{k_1+k_2+k_3+k_4}$$
$$= 1 - e^{-(k_1+k_2+k_3+k_4)t/\lambda} \quad ,$$

where C_{ij} refers to component j, j = 1, ..., k_i, in the ith subsystem. The result is an exponential distribution with parameter $\frac{\lambda}{(k_1 + k_2 + k_3 + k_4)}$. Thus, each component should have a mean life of $(k_1 + k_2 + k_3 + k_4) \times 10$ yr for the heart to have a mean life of 10 yr. As there are 99 components in the artificial heart, the requirement is an incredible mean life of 990 yr each.

Of those 99 components, 53 are in the first subsystem, 37 in the second, 5 in the third, and 4 in the fourth. If it is assumed that each component has the same exponential distribution, the mean life required of each subsystem with k_i components is

$$\frac{10 \sum_{i=1}^{4} k_i}{k_i}$$

years. That is,

$$P\left(T_{SS_i} \le t\right)$$

$$= P \text{ (at least one component in subsystem } i \text{ fails before time } t)$$

$$= 1 - P\left(C_{i1} > t, \ldots, C_{ik_i} > t\right)$$

$$= 1 - P\left(C_{i1} > t\right), \ldots, P\left(C_{ik_i} > t\right)$$

$$= 1 - \exp\left[-t/\left(10 \sum_{i=1}^{4} k_i\right)\right]^{k_i}$$

$$= 1 - \exp\left[-k_i\, t/\left(10 \sum_{i=1}^{4} k_i\right)\right] \quad .$$

This is an exponential distribution with parameter $\frac{10 \sum_{i=1}^{4} k_i}{k_i}$. Thus, for the heart to have a mean life of 10 yr, each subsystem should have a mean life of 18.68, 26.76, 198.00, and 247.50 yr, respectively. (The life values of subsystems and components are rounded to two decimal places throughout this report.) The latter values, especially, are probably impossible to achieve in practice.

3.2. The Normal Failure Distribution

When each subsystem has a normal failure distribution with parameters μ and σ^2, the probability of a failure in the system within t years is

$$P(T_S \le t) = 1 - \left(\int_{\frac{t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2}\, dz\right)^4 \quad .$$

In determining the mean life required of each subsystem given 10-yr mean life of the heart, there is the problem that the above result is not normally distributed. In an attempt to solve the problem, additional constraints are imposed and a trial and error process is used. Given preassigned values for the standard deviation of T_{SS}, where the subscript SS refers to subsystem, the following values for the mean of T_{SS}, $\mu = E(T_{SS})$, give $P(T_S \le 10) = 0.5$. Note that, by definition, the value of t such that $P(T < t) \le 0.5$ and $P(T \le t) \ge 0.5$ is called the median of the distribution of the random variable T. Hence, in satisfying the assumption that $P(T_S \le 10) = 0.5$, DBER's objective of developing an artificial heart with a mean life of 10 yr changes to that of developing a heart with a median life of 10 yr (Appendix B). Notationally, t_M denotes the median.

For $\sigma = SD(T_{SS}) = 1$, $\mu = E(T_{SS}) = 11.00$; for $\sigma = 3$, $\mu = 12.99$; for $\sigma = 5$, $\mu = 14.99$; for $\sigma = 7$, $\mu = 16.99$; and for $\sigma = 10$, $\mu = 19.98$.

If the four subsystems have 99 components and all the components have the same normal failure distribution, a comparable trial and error process involving preassigned standard deviation values is used to determine the mean life values of the components indirectly. Again, it is assumed that the median life of the heart is 10 yr. The probability of a failure in the heart within t years, given its composition of 99 components, is

$$P(T_S \le t) = 1 - \left[\int_{\frac{t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\, dz\right]^{99} \quad .$$

For $t = 10$ and $\sigma = SD(T_c) = 1, 3, 5, 7,$ and 10 where the subscript c indicates components, this equation is set equal to 0.5 and solved for μ. For these five cases, the results show that each of the 99 components should have a mean life, μ, of 12.46, 17.38, 22.29, 27.21, and 34.58 yr, respectively.

The median life required of each subsystem with k_i components each of which has the same normal distribution is calculated using

$$P\left(T_{SS_i} \le t\right) = 1 - \left[\int_{\frac{t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)dz\right]^{k_i} .$$

For subsystem I, for example, solve for t using

$$P\left(T_{SS_1} \le t\right) = 1 - \left[\int_{\frac{t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)dz\right]^{53} = 0.5 .$$

If each component has a mean life, μ, of 12.46 yr and a standard deviation, σ, of 1 yr, $P\left(T_{SS_1} \le 10.23\right) = 0.5$. That is, subsystem I has a median life of 10.23 yr. Results of using the other previously given values for the mean and standard deviation of the components of each of the four subsystems are shown in Table VII.

Still another approach may be taken. If each subsystem is required to have the same normal distribution, specifically, the same median life, then the mean life of each component in a particular subsystem can be calculated using

$$P\left(T_{SS_i} \le t\right) = 1 - \left[\int_{\frac{t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)dz\right]^{k_i} = 0.5 .$$

To illustrate, for subsystem I, setting $\sigma = SD(T_c) = 1$ and using the previously calculated value $E(T_{SS}) = 11.00 = t$ gives

$$P\left(T_{SS_1} \le 11.00\right) = 1 - \left[\int_{\frac{11.00-\mu}{1}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)dz\right]^{53} = 0.5 .$$

Trial and error give $\mu = E(T_c) = 13.23$. Similar calculations for other values of $SD(T_c)$ and $E(T_{SS})$ for each of the four subsystems are summarized in Table VIII.

3.3 The Lognormal Failure Distribution

If each subsystem has a lognormal failure distribution with parameters μ and σ^2, the probability of a failure in the heart within t years is

$$P\left(T_S \le t\right) = 1 - \left[\int_{\frac{\log_e t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)dz\right]^{4} .$$

Again, through trial and error, μ and σ are selected so that the median life of the system is 10 yr. For $\sigma = SD(T_{SS}) = 1$, the mean life required of each subsystem, $\mu = E(T_{SS})$, is 11.10 yr. If $X_{SS} = \log_e T_{SS}$ is normally distributed, $E(X_{SS}) \sim 2.40$ and $SD(X_{SS}) \sim 0.1$. For $SD(T_{SS}) = 3$, $E(T_{SS}) = 14.00$, $E(X_{SS}) \sim 2.60$, and $SD(X_{SS}) \sim 0.29$; for $SD(T_{SS}) = 5$, $E(T_{SS}) = 17.92$, $E(X_{SS}) \sim 2.77$, and $SD(X_{SS}) \sim 0.47$. Similarly, for $SD(T_{SS}) = 7$, $E(T_{SS}) = 22.93$, $E(X_{SS}) \sim 2.93$, and $SD(X_{SS}) \sim 0.63$; for $SD(T_{SS}) = 10$, $E(T_{SS}) = 32.47$, $E(X_{SS}) \sim 3.13$, and $SD(X_{SS}) \sim 0.83$.

For the heart with 99 components each having the same lognormal distribution, the probability of a failure in the system within t years is

$$P\left(T_S \le t\right) = 1 - \left[\int_{\frac{\log_e t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\,dz\right]^{99} .$$

Given $t = 10$, $P(T_S \le 10) = 0.5$, and $\sigma = SD(T_c) = 1, 3, 5, 7$, and 10, each of the 99 components should have a mean life, μ, of 12.84, 21.49, 35.71, 57.65, and 109.50 yr, respectively, for the system to have a median life of 10 yr. If $X_c = \log_e T_c$ is normally distributed, $E(X_c) \sim 2.55$, 3.02, 3.46, 3.86, and 4.35 for the five cases, and $SD(X_c) \sim 0.10$, 0.29, 0.47, 0.63, and 0.83.

In considering the distribution of the components among the four subsystems, use

$$P\left(T_{SS_i} \le t\right) = 1 - \left[\int_{\frac{\log_e t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\,dz\right]^{k_i} ,$$

where each component is assumed to have the same lognormal distribution. With this equation set equal to 0.5, the median life required of each subsystem is obtained by solving for t using the values of $\mu = E(T_c)$ and $\sigma = SD(T_c)$ calculated in the previous paragraph. Results are shown in Table IX.

If each subsystem has the same lognormal distribution (the same median life), the mean life required of each component within a particular subsystem is obtained by solving for μ using the previously calculated values for σ and t in

$$P\left(T_{SS_i} \le t\right) = 1 - \left[\int_{\frac{\log_e t-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)\,dz\right]^{k_i} = 0.5 .$$

These calculations are given in Table X.

3.4. The Gamma Failure Distribution

When each subsystem has a gamma failure distribution with parameters α and β, the probability of a failure in the system within t years is

$$P\left(T_S \le t\right) = 1 - \left(1 - \Phi\left\{\left[\left(\frac{t}{\alpha\beta}\right)^{1/3} - 1 + \frac{1}{9\alpha}\right] 3\sqrt{\alpha}\right\}\right)^4 ,$$

again by use of the Wilson-Hilferty approximation. The result is not a gamma distribution. Trial and error are used to find the mean life required of each subsystem if the median life of the system is to be 10 yr. For $P(T_S \le 10) = 0.5$, $SD(T_{SS}) = 1$, say, implies that $E(T_{SS}) = 11.00$, $\alpha \sim 120.93$, and $\beta \sim 0.09$. Likewise, $SD(T_{SS}) = 3$ gives $E(T_{SS}) = 12.97$, $\alpha \sim 18.68$, and $\beta \sim 0.69$; $SD(T_{SS}) = 5$ gives $E(T_{SS}) = 14.89$, $\alpha \sim 8.87$, and $\beta \sim 1.68$; $SD(T_{SS}) = 7$ gives $E(T_{SS}) = 16.77$, $\alpha \sim 5.74$, and $\beta \sim 2.92$; and $SD(T_{SS}) = 10$ gives $E(T_{SS}) = 19.51$, $\alpha \sim 3.81$, and $\beta \sim 5.13$.

Given the 99 components each of which has the same gamma distribution, the probability of a failure in the heart within t years is

$$P\left(T_S \le t\right) = 1 - \left(1 - \Phi\left\{\left[\left(\frac{t}{\alpha\beta}\right)^{1/3} - 1 + \frac{1}{9\alpha}\right] 3\sqrt{\alpha}\right\}\right)^{99} .$$

For t = 10, $P(T_S \le 10) = 0.5$ and $SD(T_c) = 1$, 3, 5, 7, and 10, the mean life required of each component is 12.32, 16.46, 20.23, 23.78, and 28.84 yr, respectively. In these five cases, the α values are $\alpha \sim 151.83$, 30.10, 16.37, 11.54, and 8.32 and the β values are $\beta \sim 0.08$, 0.55, 1.24, 2.06, and 3.47.

To calculate the median life required of each subsystem with k_i components, set

$$P\left(T_{SS_i} \le t\right) = 1 - \left(1 - \Phi\left\{\left[\left(\frac{t}{\alpha\beta}\right)^{1/3} - 1 + \frac{1}{9\alpha}\right] 3\sqrt{\alpha}\right\}\right)^{k_i} = 0.5$$

and solve for t using the α and β values obtained for $E(T_c)$ and $SD(T_c)$ in the previous paragraph. These results are shown in Table XI.

If each subsystem has the same gamma distribution (the same median life), solving

$$P\left(T_{SS_i} \le t\right) = 1 - \left(1 - \Phi\left\{\left[\left(\frac{t}{\alpha\beta}\right)^{1/3} - 1 + \frac{1}{9\alpha}\right] 3\sqrt{\alpha}\right\}\right)^{k_i} = 0.5$$

for $\alpha\beta = E(T_c)$ using the previously calculated values of t gives the mean life required of each component within a subsystem. These results, as well as the values for $SD(T_c)$, α, and β, are given in Table XII.

3.5. The Weibull Failure Distribution

If each subsystem has a Weibull failure distribution with parameters γ and β, the probability of a failure in the system within t years is

$$P(T_S \le t) = 1 - \exp\left(-\frac{t^{\gamma}}{\beta}\right)^{4} ,$$

which is a Weibull distribution with parameters γ and β/4. Assuming that the mean life of the heart system is 10 yr and the standard deviation is 1 yr, if $\gamma \sim 12.92$ and $\beta \sim 5.78 \times 10^{13}$, the mean life and standard deviation required of each subsystem are 11.16 and 1.05, respectively. Similarly, for $SD(T_S) = 3$, $\gamma \sim 3.70$ and $\beta \sim 2.89 \times 10^4$ imply that $E(T_{SS}) = 14.50$ and $SD(T_{SS}) = 4.37$. For $SD(T_S) = 5$, $\gamma \sim 2.10$ and $\beta \sim 6.48 \times 10^2$ imply that $E(T_{SS}) = 19.34$ and $SD(T_{SS}) = 9.68$, and for $SD(T_S) = 7$, $\gamma \sim 1.45$ and $\beta \sim 1.30 \times 10^2$ imply that $E(T_{SS}) = 25.99$ and $SD(T_{SS}) = 18.19$. If $SD(T_S) = 10$, $\gamma \sim 1.00$ and $\beta \sim 40.00$ give $E(T_{SS}) = 40.00$ and $SD(T_{SS}) = 40.00$.

For a heart with 99 components, each having the same Weibull distribution, the probability of a failure within t years is

$$P(T_S \le t) = 1 - \exp\left(-\frac{t^{\gamma}}{\beta}\right)^{99} ,$$

which is a Weibull distribution with parameters γ and β/99. A mean life of 10 yr and a standard deviation of 1, 3, 5, 7, and 10 yr for the heart system were the conditions placed on the following calculations. For $\gamma \sim 12.05$ and $\beta \sim 1.84 \times 10^{14}$, the mean life and standard deviation of each component are 14.64 yr and 1.48 yr, respectively. For $SD(T_{SS}) = 3$, $\gamma \sim 3.70$ and $\beta \sim 7.14 \times 10^5$ imply that $E(T_c) = 34.54$ and $SD(T_c) = 10.40$.

Similarly, for $SD(T_{SS}) = 5$, $\gamma \sim 2.10$, $\beta \sim 1.60 \times 10^5$, $E(T_c) = 89.20$, and $SD(T_c) = 44.65$. For $SD(T_{SS}) = 7$, $\gamma \sim 1.47$, $\beta \sim 3.39 \times 10^4$, $E(T_c) = 231.98$, and $SD(T_c) = 160.89$. For $SD(T_{SS}) = 10$, $\gamma \sim 1.01$, $\beta \sim 1.02 \times 10^3$, $E(T_c) = 966.00$, and $SD(T_c) = 959.41$.

For a subsystem with k_i components each of which has the same Weibull distribution, the probability of a failure in the subsystem within t years is

$$P\left(T_{SS_i} \leq t\right) = 1 - \exp\left(- \frac{t^\gamma}{\beta}\right)^{k_i} ,$$

which is a Weibull distribution with parameters γ and β/k_i. Given the values for $E(T_{SS})$ and $SD(T_{SS})$ obtained previously, the mean life, standard deviation, α, and β values required of each component within a subsystem, assuming an identical Weibull failure distribution for each subsystem, are as shown in Table XIII.

3.6. Summary: The Life Requirements of a Subsystem with k Components

In comparing the mean life or median life of a subsystem or component as calculated for the various probability distributions, it appears that stringent assumptions and requirements are sometimes needed.

The means and standard deviations of the exponential and Weibull distributions are extremely high. The results for the other three distributions, in similar preassigned standard deviation conditions, show comparable mean life values for the three subsystems. As for the mean life values of the 99 components, the lowest requirements came from using the gamma distribution, whereas the highest came from using the Weibull distribution.

Assuming that each component of the heart system has the same failure distribution gives median life values of each subsystem with k components derived using a normal distribution, a lognormal distribution, and a gamma distribution which are very similar to each other. Note that the median life required of a subsystem with fewer components is a few years longer than that of a subsystem with more components; i.e., the fewer the components, the less likelihood of a failure in the system, so the longer the median life. Just the opposite is true if each subsystem is assumed to have the same failure distribution, in particular, the same median life. Then, the fewer the components in the subsystem, the shorter the mean life required of each component.

It is recommended that experimental data obtained by testing the heart system be used to suggest failure distributions and parameter values of practical interest in calculating the probability of a failure within t years and the mean or median life required of a particular subsystem or component. Although these computations will be more difficult, the results will be more realistic.

REFERENCES

1. "The Evaluation of Practicability of a Radioisotope Thermal Converter for an Artificial Heart Device, Phase II - Final Report," Westinghouse Astronuclear Laboratory, Pittsburgh, Pennsylvania, Report COD-3043-10, (1973).

2. William L. Hays, Statistics (Holt, Rinehart and Winston, Inc., New York, 1963).

3. Norman L. Johnson and Samuel Kotz, Continuous Univariate Distributions 1 (Houghton Mifflin Co., Boston, 1970).

APPENDIX A
SYSTEM RELIABILITY

Because the heart is assumed to be a simple series system of components,

$$R_S = R_1, R_2, \ldots, R_n ,$$

where R_S is the system reliability and R_n is the reliability of the individual components. Given the apportioned reliability of the components as shown in Table 5.4 of Ref. 1, the reliability of the system is 0.500636. Of the 99 heart components, 22 have a reliability of 0.999999, 70 have a reliability of 0.999, and 7 have a reliability of 0.915. If these 7 components had a reliability of 0.999 instead of 0.915, the reliability of the system would be 0.925834. To speculate further, if each component had a reliability of 0.9999, 0.99999, and 0.999999, respectively, the corresponding system reliabilities would be 0.990148, 0.999010, and 0.999901.

APPENDIX B
THE MEAN VS THE MEDIAN

Two measures of a distribution's central tendency are the mean and the median. The mean is the long-term average of a random variable, and the median is the point below which 50% of the distribution lies. Although the median is mathematically less tractable, it is useful in descriptive statistics because it represents the typical score. The mean, on the other hand, is useful when making inferences beyond the sample.[2]

If the distribution is symmetrical, the mean and median are equal. If the distribution is asymmetrical or skewed, they usually are unequal. If a distribution is skewed to the right, or positively, the mean is usually larger than the median. If a distribution is skewed to the left, or negatively, the median is usually larger than the mean.

Note that the mean is very sensitive to changes in data scores at the extremes of a distribution, whereas such changes do not affect the median as long as the rank order of the scores is preserved.

TABLE I

THE PROBABILITY OF A FAILURE IN THE HEART SYSTEM FOR t YEARS ASSUMING A MEAN LIFE OF 10 YEARS, A VARIANCE OF 100 YEARS, AND THE EXPONENTIAL FAILURE DISTRIBUTION

Years	Exponential
1	0.0952
2	0.1813
3	0.2592
4	0.3297
5	0.3935
6	0.4512
7	0.5034
8	0.5507
9	0.5934
10	0.6321
11	0.6671
12	0.6988
13	0.7275
14	0.7534
15	0.7769

TABLE II

THE PROBABILITY OF A FAILURE IN THE HEART SYSTEM FOR t YEARS ASSUMING A MEAN LIFE OF 10 YEARS, A STANDARD DEVIATION OF 1 YEAR, AND VARIOUS FAILURE DISTRIBUTIONS

Years\Distribution	Normal	Lognormal	Gamma	Weibull
1	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0000	0.0000
4	0.0000	0.0000	0.0000	0.0000
5	0.0000	0.0000	0.0000	0.0001
6	0.0000	0.0000	0.0000	0.0012
7	0.0013	0.0002	0.0004	0.0078
8	0.0228	0.0144	0.0171	0.0390
9	0.1587	0.1571	0.1582	0.1534
10	0.5000	0.5199	0.5133	0.4508
11	0.8413	0.8426	0.8418	0.8517
12	0.9772	0.9698	0.9721	0.9959
13	0.9987	0.9963	0.9972	1.0000
14	1.0000	0.9997	0.9998	1.0000
15	1.0000	1.0000	1.0000	1.0000

TABLE III

THE PROBABILITY OF A FAILURE IN THE HEART SYSTEM FOR t YEARS ASSUMING A MEAN LIFE OF 10 YEARS, A STANDARD DEVIATION OF 3 YEARS, AND VARIOUS FAILURE DISTRIBUTIONS

Years\Distribution	Normal	Lognormal	Gamma	Weibull
1	0.0013	0.0000	0.0000	0.0001
2	0.0038	0.0000	0.0000	0.0017
3	0.0098	0.0000	0.0007	0.0078
4	0.0228	0.0015	0.0057	0.0225
5	0.0478	0.0134	0.0248	0.0508
6	0.0912	0.0555	0.0714	0.0975
7	0.1587	0.1427	0.1536	0.1662
8	0.2525	0.2698	0.2687	0.2580
9	0.3694	0.4160	0.4032	0.3701
10	0.5000	0.5583	0.5398	0.4952
11	0.6306	0.6813	0.6638	0.6224
12	0.7475	0.7787	0.7663	0.7396
13	0.8413	0.8509	0.8447	0.8365
14	0.9088	0.9020	0.9009	0.9079
15	0.9522	0.9367	0.9391	0.9541

TABLE IV

THE PROBABILITY OF A FAILURE IN THE HEART SYSTEM FOR t YEARS ASSUMING A MEAN LIFE OF 10 YEARS, A STANDARD DEVIATION OF 5 YEARS, AND VARIOUS FAILURE DISTRIBUTIONS

Years\Distribution	Normal	Lognormal	Gamma	Weibull
1	0.0359	0.0000	0.0012	0.0061
2	0.0548	0.0008	0.0100	0.0260
3	0.0808	0.0104	0.0346	0.0599
4	0.1151	0.0442	0.0789	0.1068
5	0.1587	0.1091	0.1421	0.1652
6	0.2119	0.1990	0.2198	0.2327
7	0.2743	0.3019	0.3065	0.3066
8	0.3446	0.4066	0.3961	0.3842
9	0.4207	0.5052	0.4839	0.4626
10	0.5000	0.5934	0.5662	0.5392
11	0.5793	0.6693	0.6407	0.6120
12	0.6554	0.7331	0.7063	0.6791
13	0.7257	0.7857	0.7627	0.7394
14	0.7881	0.8286	0.8102	0.7922
15	0.8413	0.8631	0.8497	0.8374

TABLE V

THE PROBABILITY OF A FAILURE IN THE HEART SYSTEM FOR t YEARS ASSUMING A MEAN LIFE OF 10 YEARS, A STANDARD DEVIATION OF 7 YEARS, AND VARIOUS FAILURE DISTRIBUTIONS

Years\Distribution	Normal	Lognormal	Gamma	Weibull
1	0.0993	0.0004	0.0196	0.0302
2	0.1265	0.0128	0.0610	0.0805
3	0.1587	0.0558	0.1183	0.1403
4	0.1957	0.1281	0.1855	0.2050
5	0.2375	0.2171	0.2576	0.2718
6	0.2839	0.3109	0.3308	0.3385
7	0.3341	0.4017	0.4024	0.4036
8	0.3875	0.4850	0.4706	0.4660
9	0.4432	0.5592	0.5340	0.5250
10	0.5000	0.6239	0.5922	0.5799
11	0.5568	0.6796	0.6449	0.6307
12	0.6125	0.7272	0.6921	0.6770
13	0.6659	0.7677	0.7340	0.7190
14	0.7161	0.8019	0.7710	0.7567
15	0.7625	0.8309	0.8033	0.7903

TABLE VI

THE PROBABILITY OF A FAILURE IN THE HEART SYSTEM FOR t YEARS ASSUMING A MEAN LIFE OF 10 YEARS, A STANDARD DEVIATION OF 10 YEARS, AND VARIOUS FAILURE DISTRIBUTIONS

Years\Distribution	Normal	Lognormal	Gamma	Weibull
1	0.1841	0.0094	0.1013	0.0952
2	0.2119	0.0647	0.1808	0.1813
3	0.2420	0.1515	0.2552	0.2592
4	0.2743	0.2469	0.3241	0.3297
5	0.3085	0.3386	0.3876	0.3935
6	0.3446	0.4218	0.4458	0.4512
7	0.3821	0.4952	0.4988	0.5034
8	0.4207	0.5589	0.5471	0.5507
9	0.4602	0.6140	0.5909	0.5934
10	0.5000	0.6614	0.6306	0.6321
11	0.5398	0.7022	0.6665	0.6671
12	0.5793	0.7374	0.6989	0.6988
13	0.6179	0.7677	0.7282	0.7275
14	0.6554	0.7940	0.7547	0.7534
15	0.6915	0.8168	0.7786	0.7769

TABLE VII

THE MEDIAN LIFE, t_M, REQUIRED OF EACH SUBSYSTEM WITH k_i COMPONENTS ASSUMING AN IDENTICAL NORMAL FAILURE DISTRIBUTION FOR EACH COMPONENT

Subsystem \ $E(T_c)$ / $SD(T_c)$	12.46 1	17.38 3	22.29 5	27.21 7	34.58 10
I	t = 10.23	10.70	11.16	11.62	12.32
II	10.37	11.12	11.87	12.62	13.74
III	11.33	13.99	16.65	19.31	23.29
IV	11.46	14.38	17.30	20.22	24.60

TABLE VIII

THE MEAN LIFE, μ, REQUIRED OF EACH COMPONENT WITHIN A SUBSYSTEM ASSUMING AN IDENTICAL NORMAL FAILURE DISTRIBUTION FOR EACH SUBSYSTEM

Subsystem \ $E(T_{SS})$ / $SD(T_c)$	11.00 1	12.99 3	14.99 5	16.99 7	19.98 10
I	μ = 13.23	19.67	26.12	32.57	42.25
II	13.08	19.25	25.41	31.58	40.83
III	12.13	16.38	20.64	24.89	31.27
IV	12.00	15.99	19.98	23.97	29.96

TABLE IX

THE MEDIAN LIFE, t_M, REQUIRED OF EACH SUBSYSTEM WITH k_i COMPONENTS ASSUMING AN IDENTICAL LOGNORMAL FAILURE DISTRIBUTION FOR EACH COMPONENT

Subsystem \ $E(T_c)$ / $SD(T_c)$	12.84 1	21.49 3	35.71 5	57.65 7	109.50 10
I	t = 10.23	10.71	11.16	11.58	12.13
II	10.38	11.16	11.93	12.66	13.65
III	11.42	14.77	18.74	23.15	30.25
IV	11.57	15.35	19.93	25.15	33.73

TABLE X

THE MEAN LIFE, μ , REQUIRED OF EACH COMPONENT WITHIN A SUBSYSTEM ASSUMING AN IDENTICAL LOGNORMAL FAILURE DISTRIBUTION FOR EACH SUBSYSTEM

Subsystem						
	$E(T_{SS})$	11.10	14.00	17.92	22.93	32.47
	$SD(T_c)$	1	3	5	7	10
I	μ =	13.86	26.90	51.28	93.53	207.22
II		13.67	25.81	47.96	85.51	184.12
III		12.43	19.49	30.54	46.77	83.11
IV		12.26	18.76	28.71	43.06	74.53

TABLE XI

THE MEDIAN LIFE, t_M, REQUIRED OF EACH SUBSYSTEM WITH k_i COMPONENTS ASSUMING AN IDENTICAL GAMMA FAILURE DISTRIBUTION FOR EACH COMPONENT

Subsystem						
	$E(T_c)$	12.32	16.46	20.23	23.78	28.84
	$SD(T_c)$	1	3	5	7	10
	α	151.83	30.10	16.37	11.54	8.32
	β	0.08	0.55	1.24	2.06	3.47
I	t =	10.20	10.51	10.74	10.94	11.19
II		10.32	10.83	11.22	11.54	11.96
III		11.20	13.14	14.76	16.19	18.10
IV		11.33	13.48	15.29	16.90	19.08

TABLE XII

THE MEAN LIFE, STANDARD DEVIATION, α, AND β VALUES REQUIRED OF EACH COMPONENT WITHIN A SUBSYSTEM ASSUMING AN IDENTICAL GAMMA FAILURE DISTRIBUTION FOR EACH SUBSYSTEM

Subsystem \ $E(T_{SS})$		11.00	12.97	14.89	16.77	19.51
I	$E(T_c)$ =	13.12	19.02	24.67	30.19	38.30
	$SD(T_c)$ ~	1.00	3.00	5.00	7.00	10.00
	α ~	172.19	40.18	24.35	18.60	14.67
	β ~	0.08	0.47	1.01	1.62	2.61
II		12.99	18.67	24.14	29.47	37.30
		1.00	3.00	5.00	7.00	10.00
		168.86	38.74	23.30	17.72	13.91
		0.08	0.48	1.04	1.66	2.68
III		12.12	16.28	20.36	24.37	30.29
		1.00	3.00	5.00	7.00	10.00
		146.82	29.46	16.59	12.13	9.17
		0.08	0.55	1.23	2.01	3.30
IV		11.99	15.94	19.82	23.64	29.28
		1.00	3.00	5.00	7.00	10.00
		143.85	28.24	15.72	11.41	8.57
		0.08	0.56	1.26	2.07	3.42

TABLE XIII

THE MEAN LIFE, STANDARD DEVIATION, α, AND β VALUES REQUIRED OF EACH COMPONENT WITHIN A SUBSYSTEM ASSUMING AN IDENTICAL WEIBULL FAILURE DISTRIBUTION FOR EACH SUBSYSTEM

Subsystem						
	$E(T_{SS})$	11.16	14.50	19.28	25.99	40.00
	$SD(T_{SS})$	1.05	4.37	9.77	18.19	40.00
I	$E(T_C)=$	15.48	42.45	128.57	420.22	2158.18
	$SD(T_C)\sim$	1.57	12.80	64.53	301.22	2171.16
	α ~	11.98	3.69	2.09	1.41	0.99
	β ~	3.00×10^{14}	1.50×10^{6}	3.34×10^{4}	5.88×10^{3}	2.06×10^{3}
II		14.98	38.48	108.43	297.41	1480.00
		1.50	11.56	54.50	201.88	1480.00
		12.13	3.71	2.09	1.50	1.00
		3.00×10^{14}	1.10×10^{6}	2.30×10^{4}	6.00×10^{3}	1.48×10^{3}
III		12.64	22.41	41.61	78.83	200.00
		1.19	6.75	20.88	55.31	200.00
		12.94	3.70	2.09	1.45	1.00
		3.00×10^{14}	1.45×10^{5}	3.16×10^{3}	6.40×10^{2}	2.00×10^{2}
IV		12.42	21.13	37.39	67.56	160.00
		1.16	6.34	18.77	47.29	160.00
		13.03	3.71	2.09	1.45	1.00
		3.00×10^{14}	1.20×10^{5}	2.51×10^{3}	5.21×10^{2}	1.60×10^{2}

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